Tips

There are many techniques and concepts that can be used in solving Path Puzzles. Each tip in this section covers a different technique.

A tip for using tips.
The way to apply a tip to a particular puzzle will not always be obvious. The tip may be useful when you first start a puzzle or later, when a few things are filled in.

There are practice puzzles throughout this section. All practice puzzles have solutions in the back of the book.

Tip #1
You Can Do This!

Some Path Puzzles can look pretty tough, but ***you can* solve them!** Here are some techniques you can use:

Look over the whole puzzle and think about the meaning of each clue. Think about the general shape of the path and where it has to go.

Choose a cell to focus on. What happens if the path goes through that cell? What if it doesn't?

Use notation. Any time you figure out anything at all, write it into the puzzle somehow. Then, think, "What does this new information imply?" Suggestions for notation are in "Solving Step-by-step" on page 11.

Don't be intimidated by the size of a puzzle. Large doesn't always mean difficult.

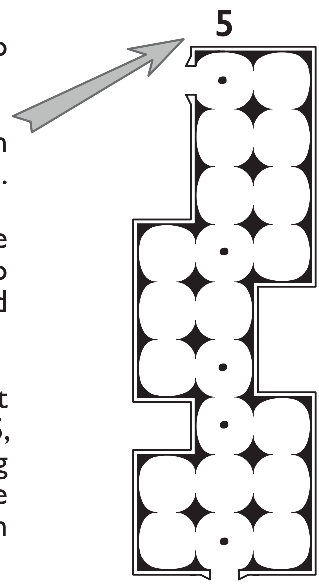
Get help by email. Yes, you can email a Path Puzzle expert for help! Send an email to help@pathpuzzles.com. Include a photo of the state of your path puzzle or just the page number and puzzle you're working on. We'll help you get through it.

Tip #2
Making the Most of the Clues You Get

Sometimes, there will not seem to be enough information to solve a puzzle. In this situation, focus on what you *do* know rather than what you don't. Here's an example:

There doesn't seem to be much to work with here.

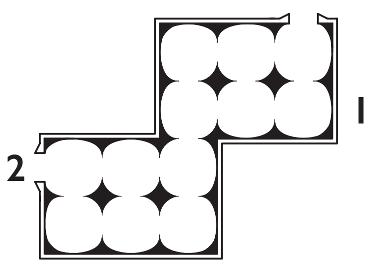
It's okay. There's a lot of information in the shape of the grid and in this 5.

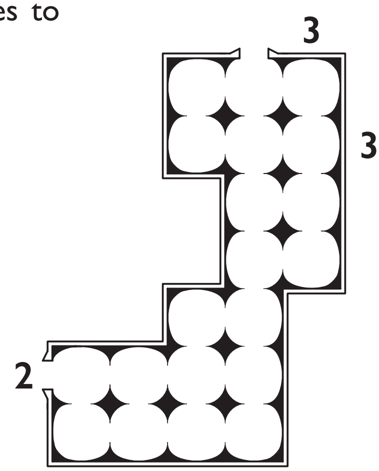


Knowing only the shape of the grid, what cells *must* the path to go through? All such cells are marked with dots.

Since there are five cells that must be used in the column under the 5, you cannot use any of the remaining cells in that column. Cross out the empty cells below the 5 and a path will emerge.

Here are a couple more puzzles to practice on.





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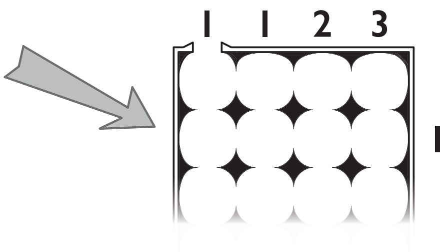
Tip #3
What Goes In, Must Come Out.

For every place the path goes, it needs a way in and a way out. Put another way, if the path appears in a cell, it must also appear in two adjacent cells. (An exception is if the cell in question is next to one of the doors on the edges of the grid.)

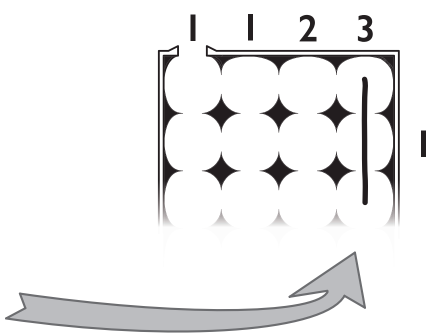
Consider this situation. Only one cell from this row may be used.

This means that whatever cell is used, the path cannot enter or exit that cell to the right or left, as that would use at least two cells from the row.



Instead, the path must pass through the row vertically. This uses three vertically connected cells.

The only column that can have three cells of the path in it is this one.

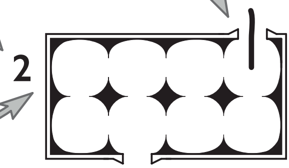


On the next page is another way this idea can be used.

Note that this cell is needed to enter the grid.

Also note that this 2 allows only one additional cell to be used in the top row.

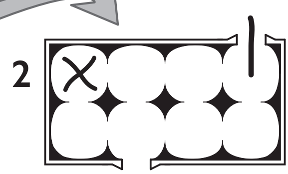
So what happens if we try to use this corner cell?



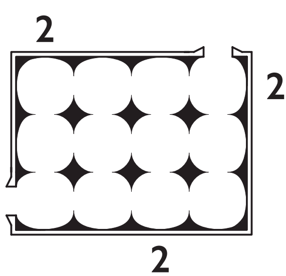
If the path goes to this cell, it must also visit the two adjacent cells. This causes the path to use three cells in a row that can only contain two.

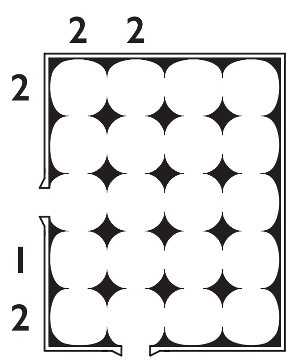
Therefore, the path may not use the corner cell.

Furthermore, once the corner is eliminated, the cell to its right becomes a new corner. It can now be ruled out in the same way.



Here are some puzzles in which this principle can be used several times.





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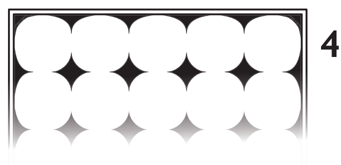
Tip #4
"The Almost-full Edge Principle"

Here's a handy idea you can use when you have one more cell than you need on the edge of a Path Puzzle.

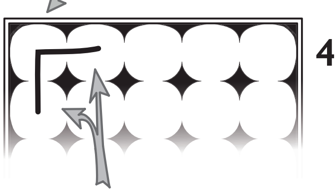
Notice that the top edge of the puzzle has ***five cells***...

...and ***four*** of them are used in the solution.



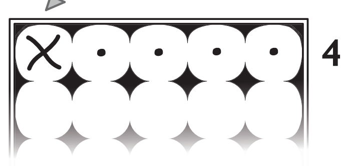
Now, consider the two options below for this corner cell.

Option #1
The path passes through this cell.



In this case, the path must also pass through both of these cells.

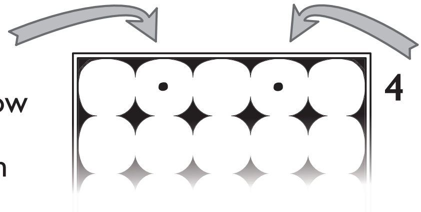
Option #2
The path *does not* pass through this cell.



In this case, you must use all four other cells in the row to satisfy the "4"

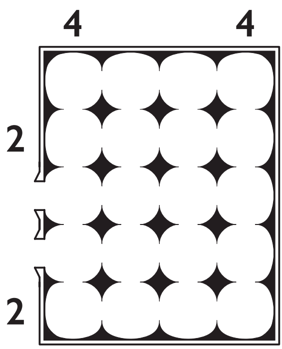
Before reading on, do you see the conclusion you can make here?

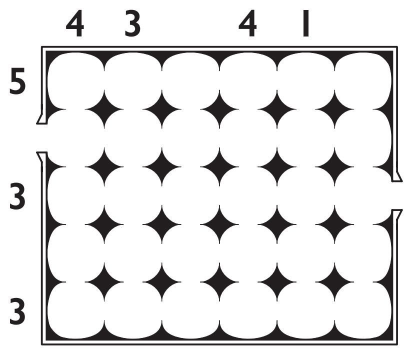
Conclusion: With either option, *you must use this cell!* Mark the cell to show that that the path will pass though it in some way.

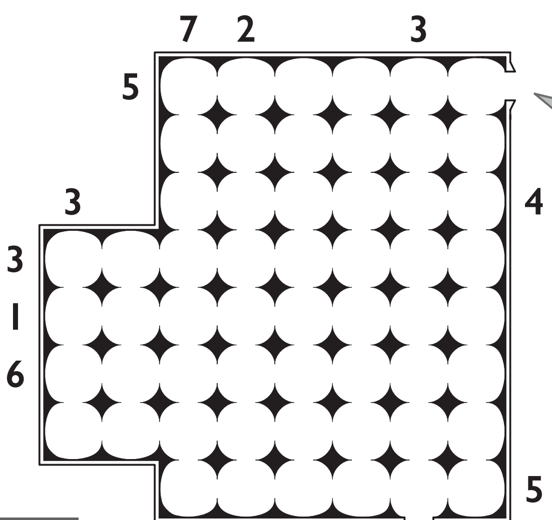


The principle applies to this cell in the same way.

As you solve these practice puzzles, look for every opportunity to use this idea, not just on the outer edges. Every time you notice an opportunity, think through the logic of options #1 and #2 from the previous page to make sure it works.







How does this door affect the use of this idea?

Work through options one and two and from the previous page to find out.

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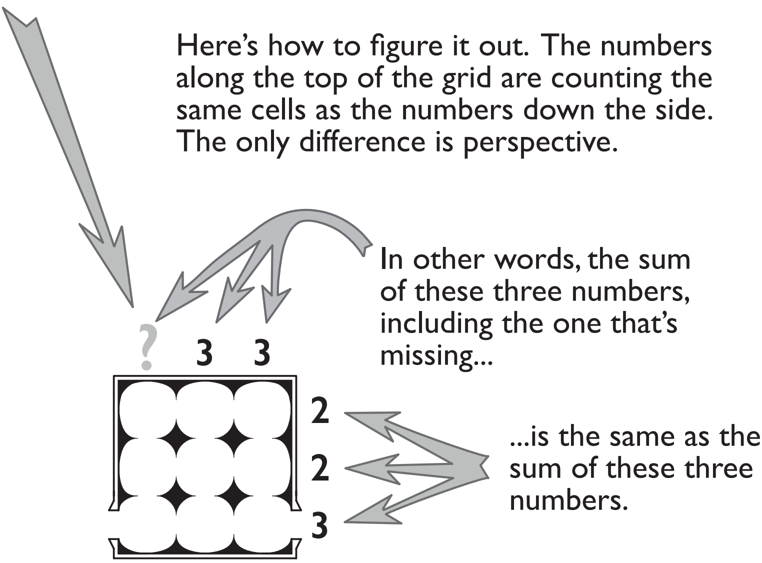
Tip #5
Solving for Missing Numbers

Sometimes, it can be useful to figure out the value of a numerical clue that is not given.

If there were a number here, what number would it be?



So, ?+3+3 = 2+2+3 Simplify a bit to get: ?+6 = 7 Now we can see that the missing number has to be a 1.

If more than one number is missing, this method may not always give you the numbers you want directly. Sometimes, it will only narrow the possibilities to a range of numbers. You'll have to figure it out from there.

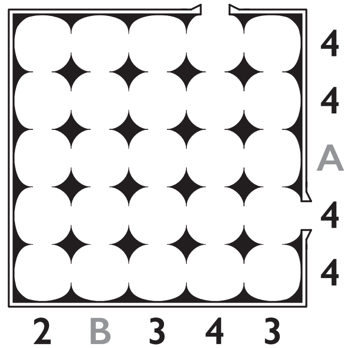
On the next page are some puzzles for practicing this technique.

For easy reference, the missing numbers are called “A” and “B.” The equation for figuring out their values is:

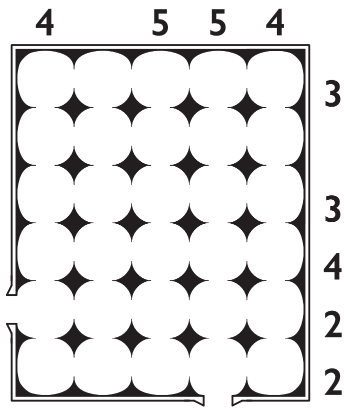
$$4+4+4+4+A = 2+3+4+3+B$$

The size of the grid means that the missing numbers must be in the range 0 through 5. Can you figure out what they are?

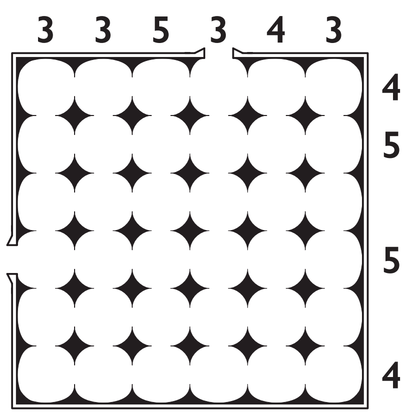
The equation should give you two possibilities. You’ll have to think about the shape of the path to choose correctly.



In this puzzle, as in the puzzle above, using this technique will narrow the possibilities. You’ll have to take it from there



Here, you can solve for the two missing row clues. After solving for the two numbers, you’ll have to figure out which one goes where.



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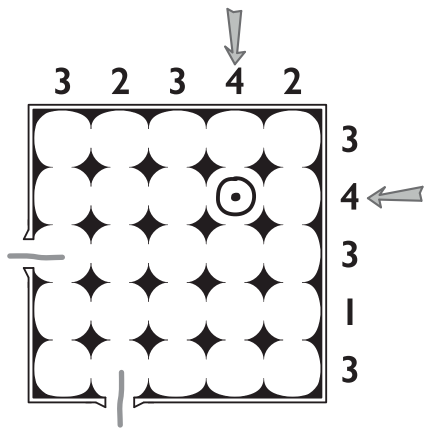
Tip #6
Proof by Contradiction

The mathematician G.H. Hardy described *Proof by Contradiction* as "one of a mathematician's finest weapons". If you find yourself not knowing what to try next, this technique may be helpful.

Proof by Contradiction is the practice of proving something is true by showing that the *opposite* case *cannot* be true.

In the puzzle below, there is both a column and a row with four of their five cells occupied by the path. It seems likely that the cell where this column and row intersect would contain a piece of the path. Let's put a dot there representing a piece of the path as something to try out.

You may want to circle the dot as a reminder of what you're trying to prove.

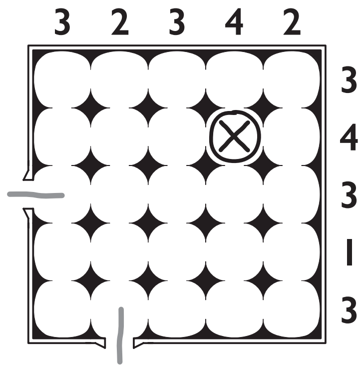


Having placed the dot, what do we do next? The hypothesis that the path goes through that spot has not made the next step any easier to see. Furthermore, we haven't actually proven that the path goes through that cell, and it's hard to know how to proceed, knowing that what we have so far might not be right.

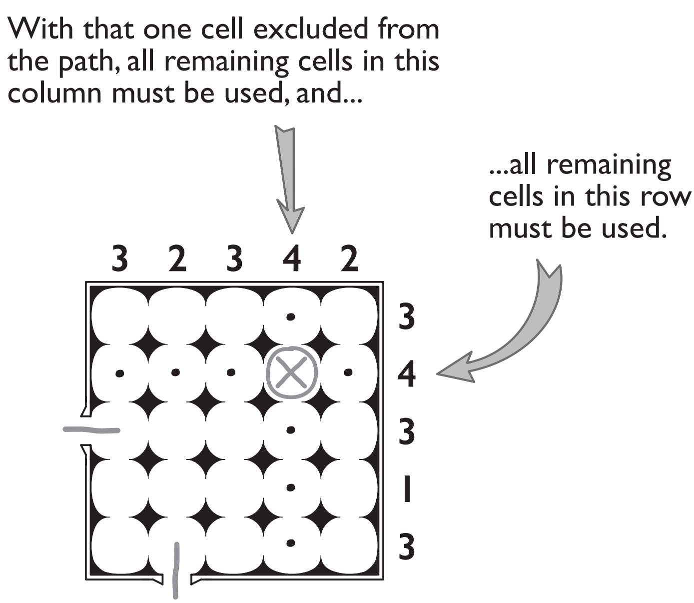
We could try adding another likely-looking assumption, but doing this only compounds the uncertainty. Sooner or later, one of these assumptions is going to be incorrect and it's going to be very difficult to know which one. *We won't be able to complete the path and we won't know why.*

On the next page, we'll try an alternate hypothesis that will help us find the path.

Instead of proposing that the path goes through that cell, let's see what happens if we propose that the path *does not* go through that cell. To notate this, we erase the dot and put an X there instead.

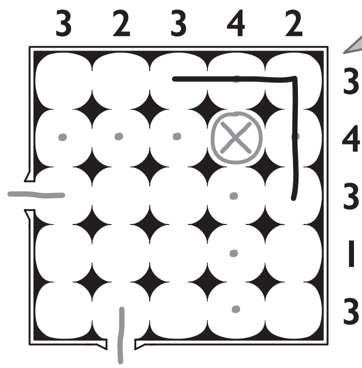


Immediately, consequences emerge.



We've filled all these cells with dots representing pieces of the path.

Now note that because there are pieces of the path above and to the right of the 'X', the path must go around it, like this.



Do you see the problem here?

The path now takes up three cells in the rightmost column. This is in conflict with the 2 at the top of the column! Our proposition that the path does not go through the cell with the 'X' in it has forced the path to be in a place where it cannot be. We've arrived at a contradiction. Therefore, our proposition is incorrect. The only other option is that the path *does* run through that cell. This is no longer an assumption. We have actually proven it by contradiction and we can proceed knowing that we are on solid ground.

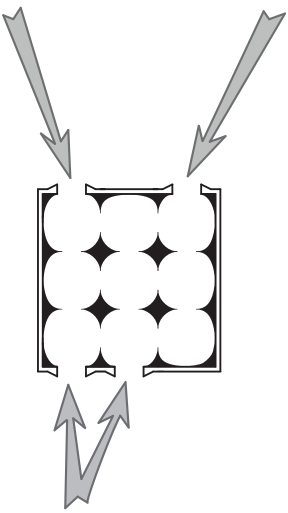
Now you can go back to the first diagram in this explanation and solve the puzzle knowing that the path goes through that cell.

Solution on page 170.

Tip #7
Using Parity to Find the Right Door

Parity is the property of a number that we refer to as “even” or “odd”. Believe it or not, you can use parity to determine which doors at the edges of a Path Puzzle are the real ones and which are false.

Try this. Find a path, *any path you like*, from this door . . . . to this door.



Count the cells in your chosen path. Now find another path using the same two doors and count the cells in it. Do this a couple more times. Notice something interesting? The number of cells is always odd!

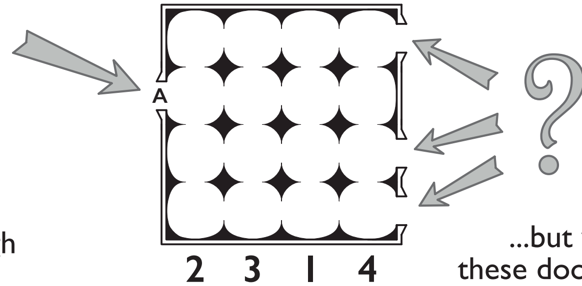
Next, try the same thing with these two doors. Notice that no matter what path you take, the number of cells will be even.

You will find that given any two cells in a grid, the parity of the number of cells required to get from one to the other is the same *no matter what path you take*.

So, how can you use parity to choose the right doors in a Path Puzzle? Read on!

Let's consider this situation:

You've figured out that this door (called "door A" for reference) is one of the two doors you must use to get through the grid...

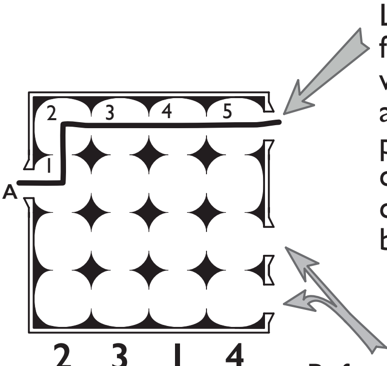


...but which of these doors is the other door in the solution?

First, determine the number of cells used by the path. For this puzzle, the clues tell you how many cells are used in each column, so the number of cells in the whole path is the sum of these four numbers. 2+3+1+4=10 There are 10 cells in the path.

10 is an even number.

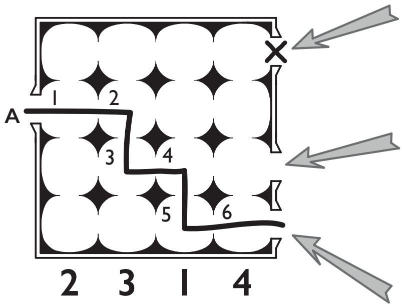
So the question is, starting at door A, which of the three doors on the right can be reached using an even number of cells?



Let's try this door. Find any path from door A to this door. The path we've chosen requires 5 cells. 5 is an odd number. From the previous page, we know there is no path we could take with an even number of cells. Therefore, this door cannot be used.

Before reading on, can you use this technique to figure out which of these two doors must be used?

Recall that we've added up the numerical clues to determine that the solution to this puzzle must use an even number of cells.



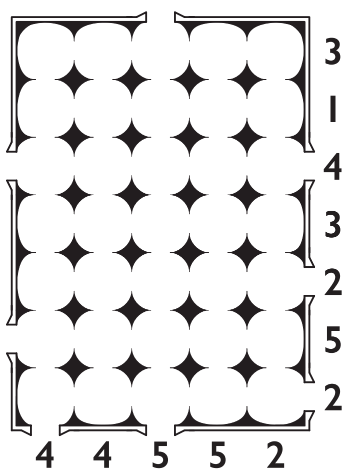
We've already seen that any path from door A to this door will use an odd number of cells, so this door cannot be used in the solution.

Likewise, this door can also only be arrived at using an odd number of cells, so it cannot be used either.

This door, however, can be arrived at from door A using a path with an even number of cells, making it the only usable door on this side of the puzzle.

Note that the path drawn is not the solution to the puzzle. It's just a test path to determine the parity of the path between doors.

When using this technique, remember you have to figure out one door to get started. (door A in the example above) In this puzzle, what does the 1 on the second row tell you about where "door A" is ?



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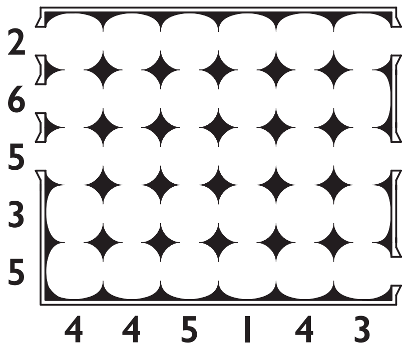
Try this one on your own. If you get stuck choosing your "Door A", read the hint below.



The 1 below the grid can tell you that one door on the left and one door on the right must be used. So which one is "Door A"—the door at the start of your test path? Note that the three doors on the right are two or four cells away from each other. Two and four are even numbers. This means that those three doors are all equivalent in terms of parity. In other words, if it takes an odd number of cells to get to one of them, it will take an odd number of cells to get to either of the other two, so *it doesn't matter which one you use in your test path.* The result will be the same.

Solution on page 171.